



A Smoothed Eclipse Model for Solar Electric Propulsion Trajectory Optimization

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Motivation

Solar electric propulsion (SEP) is the primary means of employing low-thrust on a space mission.

Eclipse constraints are discontinuous and problematic for gradient-based trajectory optimization.

Contribution

A smoothed eclipse model suitable for gradient-based trajectory optimization.

Demonstrated improvement in mass delivered from LEO to GEO using a second-order optimizer.

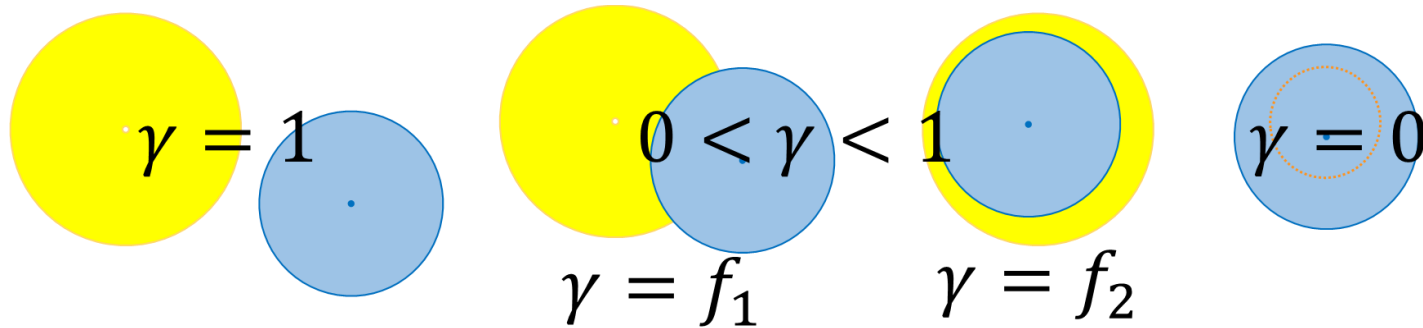


Approach

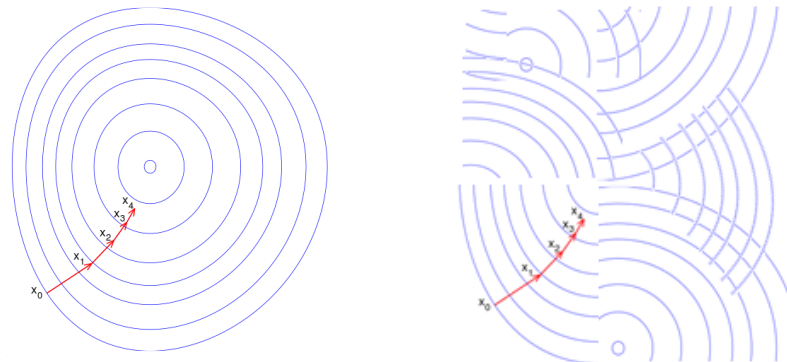
- Introduce the *sunlight fraction*, γ , to include eclipse effects

$$\text{power available} = \gamma (\text{computed power})$$

- Sunlight fraction is a piecewise function.



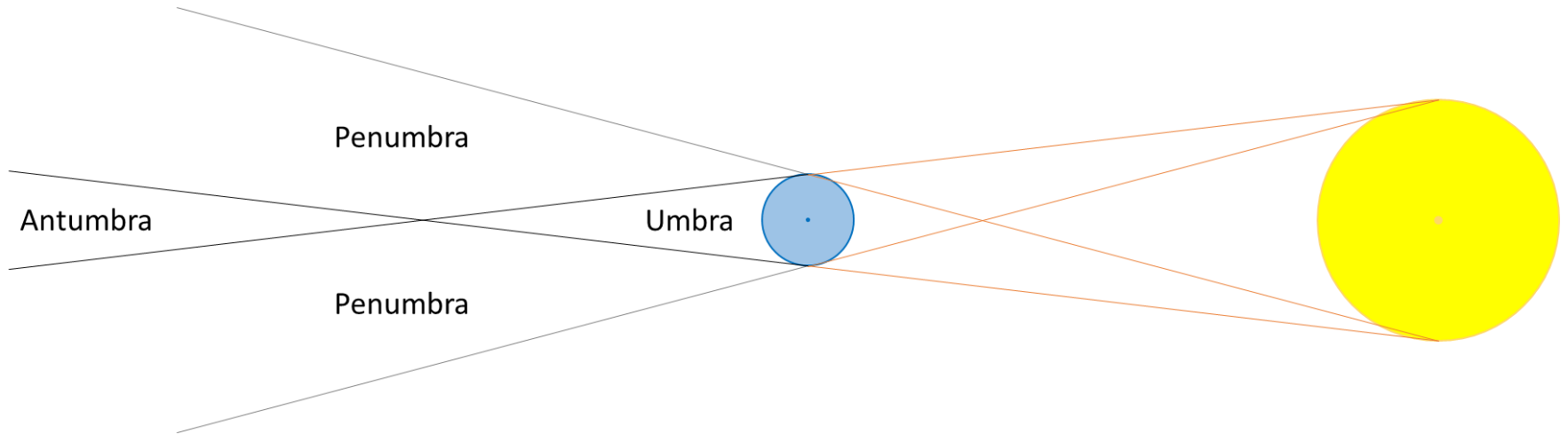
- Discontinuous derivatives are a problem for gradient-based optimization.



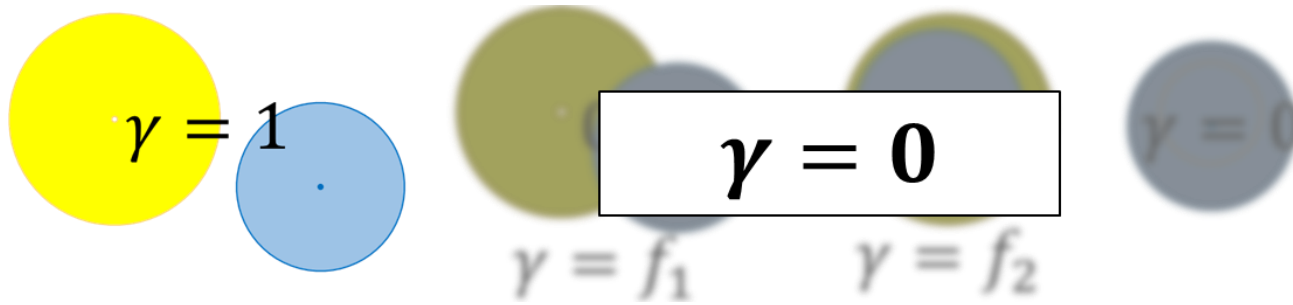


The Penumbra Constraint

- Disallow thrusting in shadow, even partial eclipse



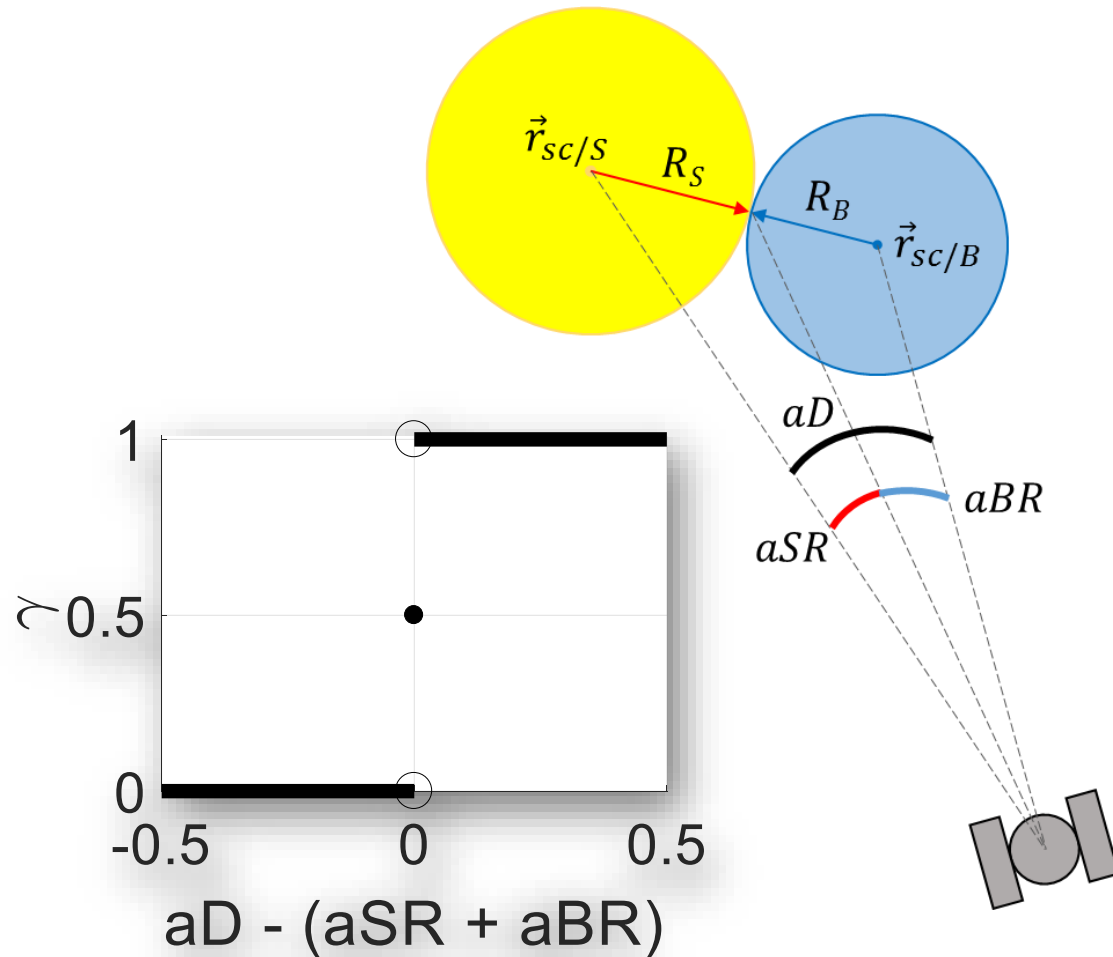
- Percent sunlight is now a step function





Heaviside Sunlight Fraction

- The Heaviside step function is half-valued at the transition.



Apparent Sun Radius

$$aSR = \sin^{-1} \frac{R_S}{r_{sc/S}}$$

Apparent Body Radius

$$aBR = \sin^{-1} \frac{R_B}{r_{sc/B}}$$

Apparent Distance

$$aD = \cos^{-1} \left(\frac{\vec{r}_{sc/B} \cdot \vec{r}_{sc/S}}{r_{sc/B} r_{sc/S}} \right)$$

Eclipse occurs when

$$aD < aSR + aBR$$



Logistic Sunlight Fraction

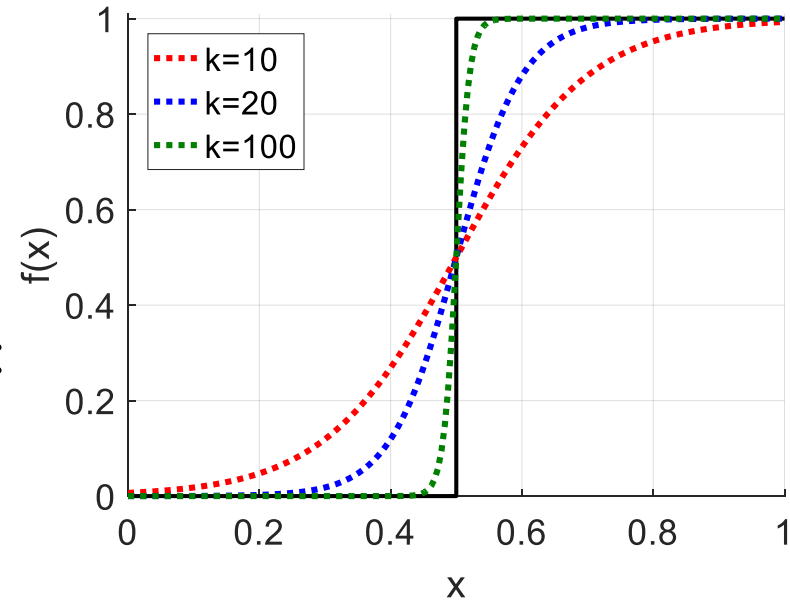
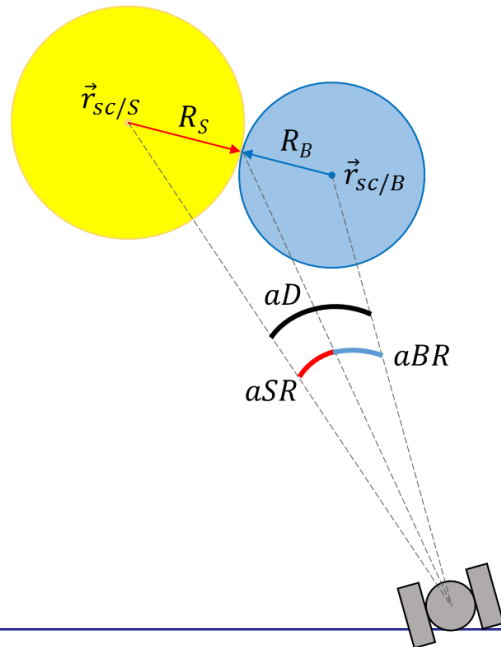
- The logistic function is a smooth approximation to the Heaviside step function,

$$f(x) = \frac{1}{1 + e^{-k(x-x^*)}}$$

and is continuously differentiable.

Transition occurs at $x = x^*$.

Eclipse occurs at $aD = aBR + aSR$:

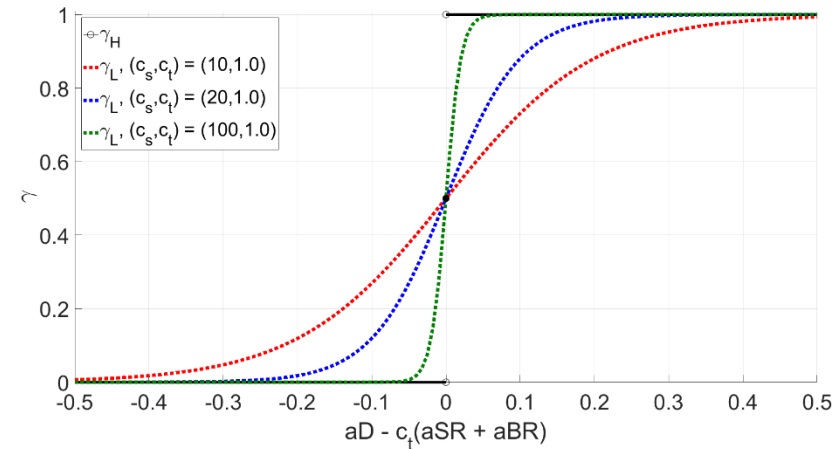
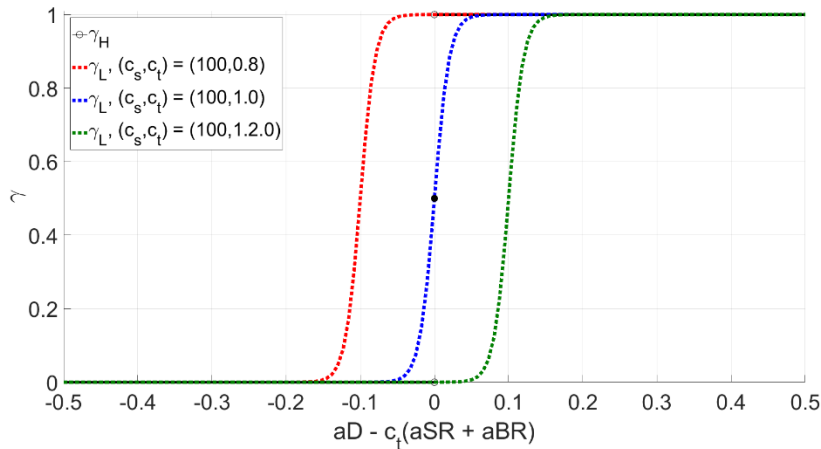


$$\gamma_L = \frac{1}{1 + e^{-c_s[aD - c_t(aSR + aBR)]}}$$



Sharpness and Transition Coefficients

- Sharpness coefficient, c_s , and transition coefficient, c_t , permit tuning of the smoothed eclipse model

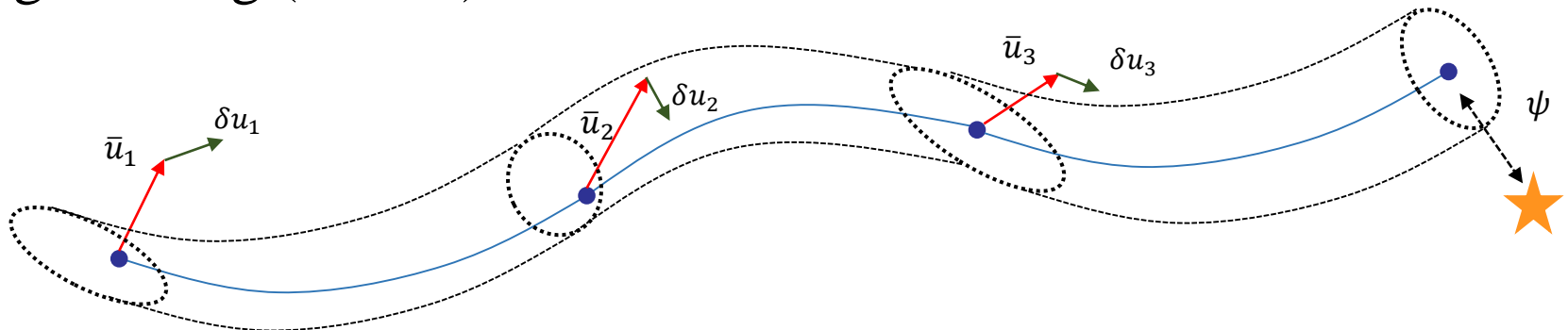


- Can select (c_s, c_t) to minimize error, numerical reasons, or operational considerations (early/late power down/up).
- For $c_t = 1.0$, find optimal $c_s \sim 289.78$ at Earth, $c_s \sim 432.35$ at Mars
 - holds across a range of spacecraft semi-major axes and eccentricities
 - central body and its heliocentric orbit are the drivers



LEO to GEO Transfer

- Reproduce transfer from Betts (2014)
 - min-fuel LEO to GEO in 248.5 revolutions
 - construct initial guess with Lyapunov control thrust arcs and forced coast arcs through eclipse
 - refine solution with Sparse Optimization Suite (SOS)
 - direct transcription and sequential nonlinear programming
- Now use smoothed eclipse model and Hybrid Differential Dynamic Programming (HDDP)



- HDDP control updates minimize local quadratic models
- 1st and 2nd derivatives of cost function and dynamics w.r.t states, controls, multipliers, parameters



Problem Setup

- Minimize propellant consumed for LEO to GEO transfer in 248.5 revolutions

Table 1.: Spacecraft Parameters.

m_0	1000 kg	T_{max}	1.445 N
I_{sp}	1849.3477486671852 s	P_0	13.1031921568649 kW

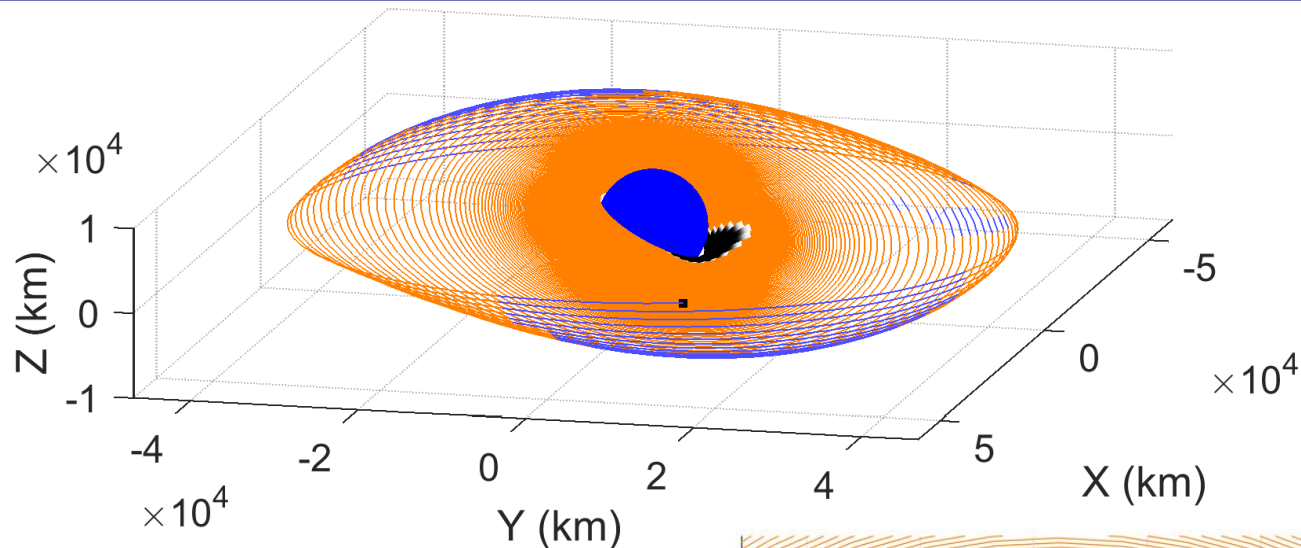
Table 2.: Initial and Target Orbit States.

p_0	6878.14 km	p_f	42241.095482827557 km
f_0	0.0	f_f	0.0
g_0	0.0	g_f	0.0
h_0	-0.25396764647494369	h_f	0.0
k_0	0.0	k_f	0.0
L_0	π		
t_0	0.0		

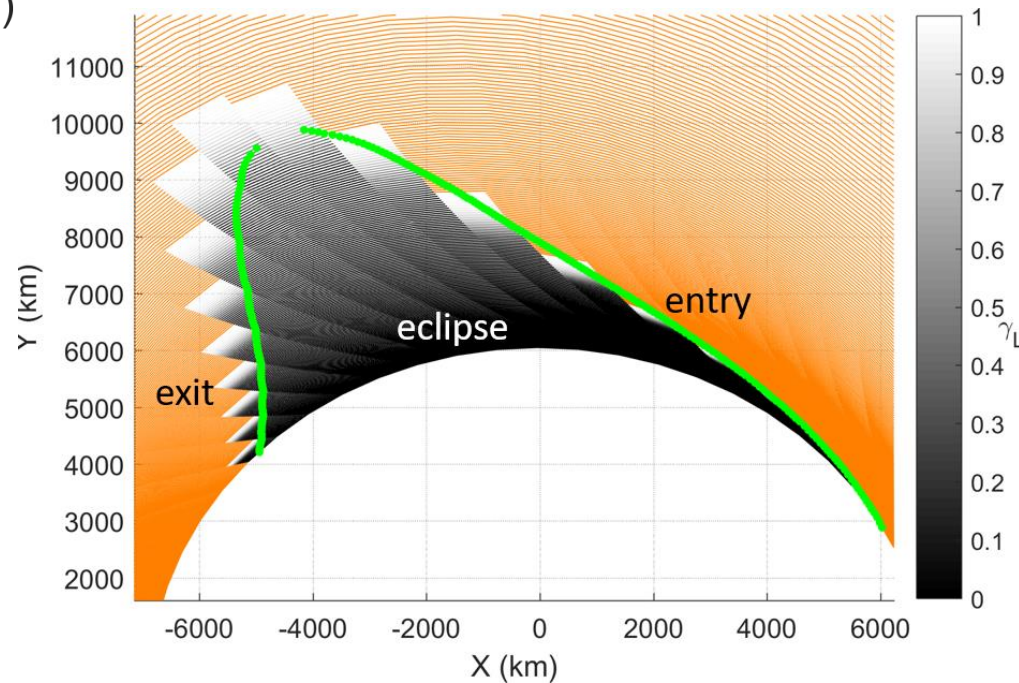
- Modified Equinoctial Elements above (500 km circular 28.5° LEO)
- Perturbations: Lunar and Solar gravity, Earth $J_2 - J_4$



Smoothed Eclipse Model Result



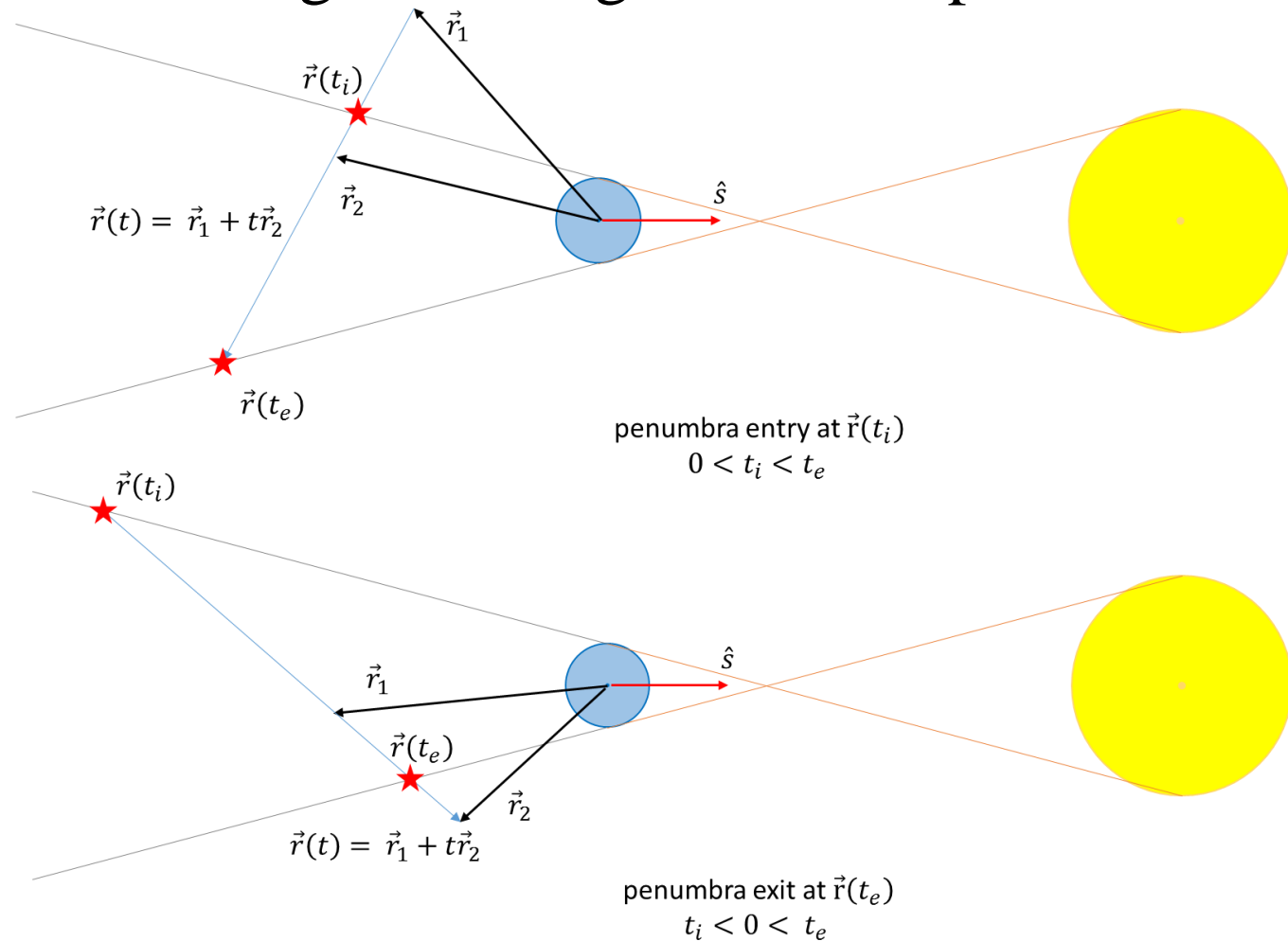
- Improved final mass to 733.29 kg from Betts' 718.79 kg
- Smoothed model eclipse detection is 'automatic', but suffers discretization error
- Violate penumbra constraint stepping through entry, and powers up late after exit





Penumbra Entry/Exit Detection

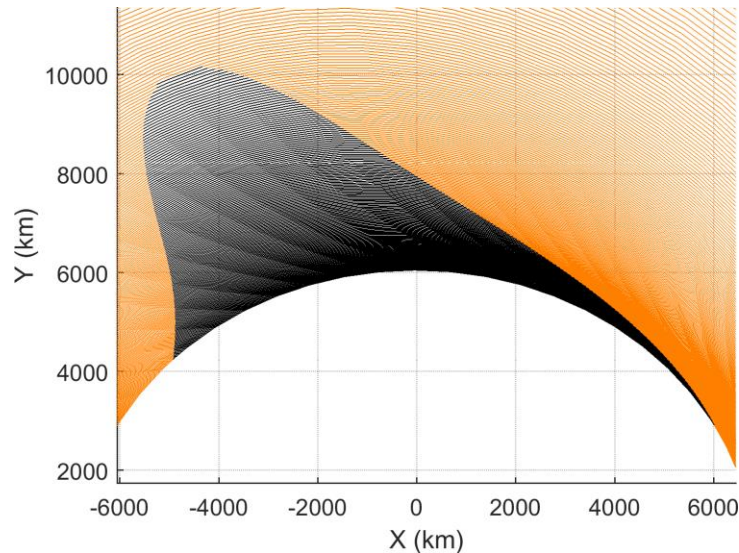
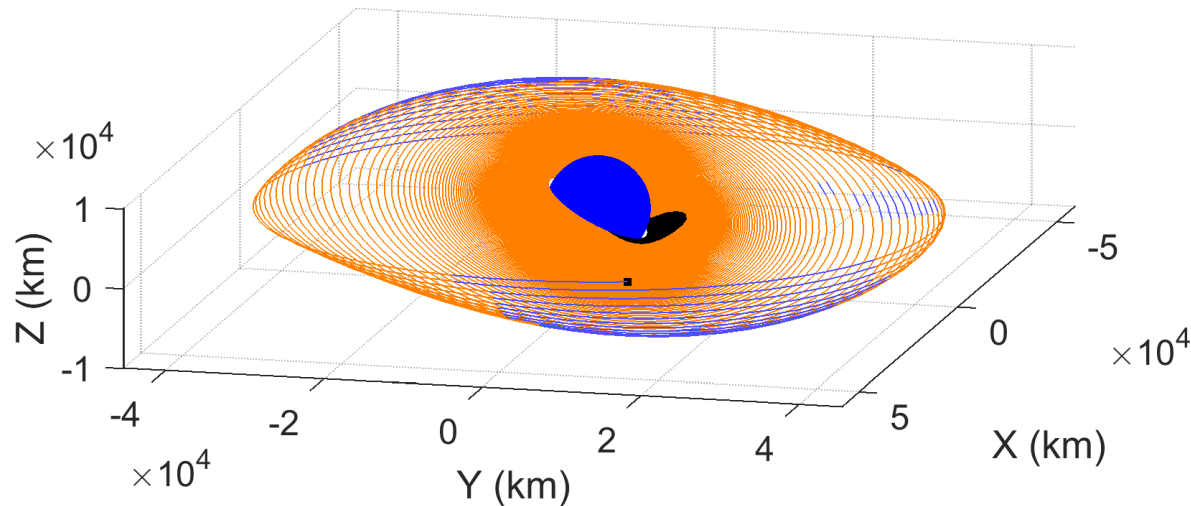
- Compute penumbra entry/exit locations as intersection of line between integration stages, and the penumbral cone





Refined Solution

- Insert integration stages at the penumbra entry/exit locations, use first solution as initial guess, assign $T_{max} = 0$ for shadow



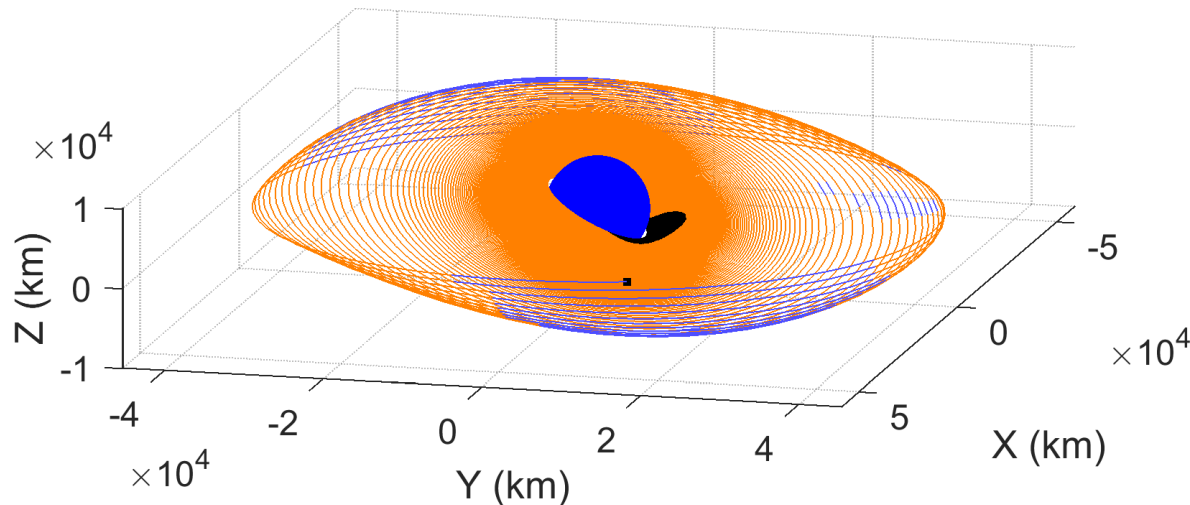
Iteration	m_f (kg)	t_f (days)
1	733.29	45.78
2	732.61	45.58
3	732.41	45.51
4	732.29	45.49
5	732.24	45.46
Betts ³⁾	718.79	43.13

Conclusion

- Smoothed Eclipse Model presented as the logistic sunlight fraction:

$$\gamma_L = \frac{1}{1 + e^{-c_s[aD - c_t(aSR + aBR)]}}$$

- Demonstrated in a second-order gradient-based trajectory optimization algorithm, HDDP
- Improved delivered mass for LEO to GEO transfer

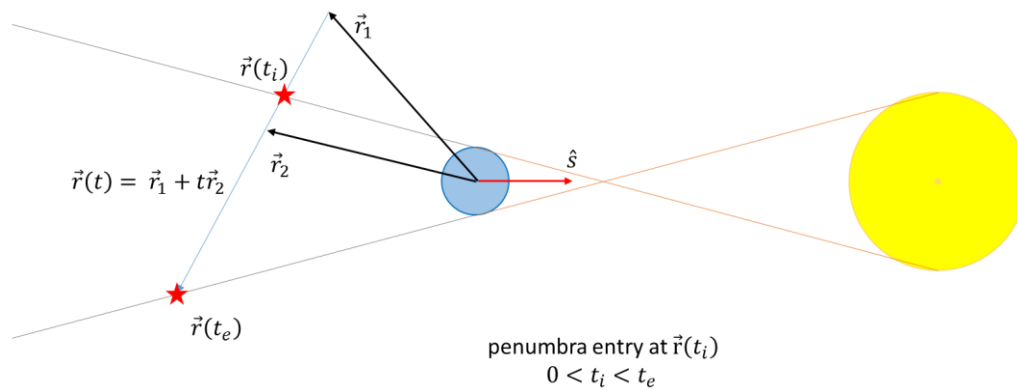


Backup Slides



Penumbra Entry/Exit Detection

- Compute penumbra entry/exit locations as intersection of line between integration stages, and the penumbral cone



$$\mathbf{r}(t) = \mathbf{r}_1 + t\mathbf{r}_2$$

$$(\mathbf{r} - \mathbf{x}_p)^T (\hat{\mathbf{s}}\hat{\mathbf{s}}^T - I \cos^2 \alpha_p) (\mathbf{r} - \mathbf{x}_p) \geq 0$$

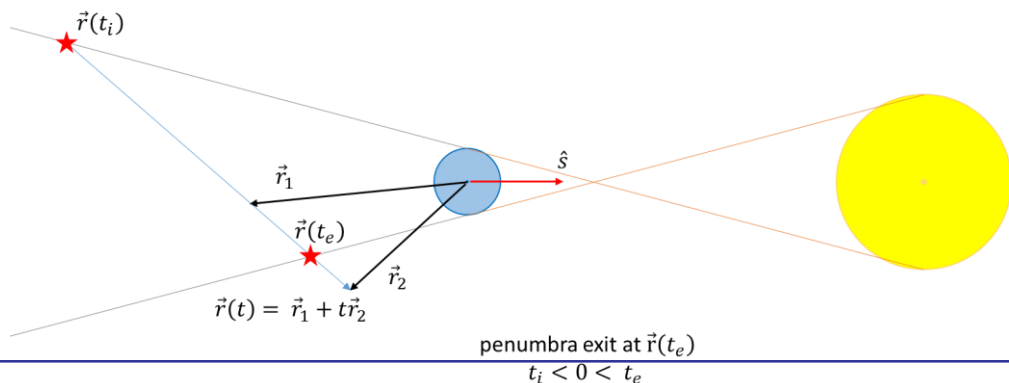
$$\mathbf{M} = (\hat{\mathbf{s}}\hat{\mathbf{s}}^T - I \cos^2 \alpha_p)$$

$$a = \mathbf{r}_2^T \mathbf{M} \mathbf{r}_2$$

$$b = \mathbf{r}_2^T \mathbf{M} (\mathbf{r}_1 - \mathbf{x}_p)$$

$$c = (\mathbf{r}_1 - \mathbf{x}_p)^T \mathbf{M} (\mathbf{r}_1 - \mathbf{x}_p)$$

$$t = (-b \pm \sqrt{b^2 - ac})/a$$





Refined Solution

- Insert integration stages at the penumbra entry/exit locations, use first solution as initial guess, assign $T_{max} = 0$ for shadow

$$\Delta\nu = \cos^{-1} \left(\frac{\mathbf{r}_e^T \mathbf{r}_1}{\|\mathbf{r}_e\| \|\mathbf{r}_1\|} \right)$$

$$\tau_e = \tau(\mathbf{X}_1, \tau_1, \Delta\nu)$$

$$\mathbf{X}_e = \mathbf{f}(\mathbf{X}_1, \tau_e - \tau_1)$$

$$\mathbf{X}_2 = \mathbf{f}(\mathbf{X}_e, \tau_2 - \tau_e)$$

